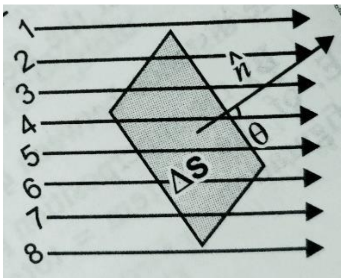
Area vector and electric flux –

As we know area is a scaler quantity , but in some cases or in case of some problems area is taken as vector quantity . the direction of area is taken as the normal of the surface at a given point . The vector associated with every area element of a closed surface is taken to be in the direction of the outward drawn normal .

Electric flux – Electric flux over an area in an electric field is the total number of field line passes through that area normally . It is denoted by ϕ its unit is Nm^2C^{-1} . And its dimension is **$[M^1L^3T^{-3}A^{-1}]$.** Electric flux is a scaler quantity.



The number of electric field lines crossing this area is proportional to field intensity E . Let electric field lines makes an angle θ with the area vector dS , and E is the electric field intensity , then we can write $d\phi = E.(dS\cos\theta)$ [since $dS\cos\theta$ is the component of area vector normal to the surface].

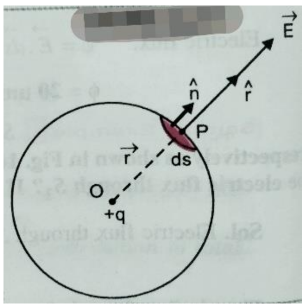
So , $d\phi = \vec{E}\,.\,\overrightarrow{dS}$

Or, $\phi_E = \int E\, dS \cos\theta$

Gauss's law in electrostatics –

According to Gauss for a closed surface total flux in the free surface is **$1/\epsilon_0$** times the total charge inside the closed surface.

Proof of Gauss's law -



Suppose a charge q is placed at the centre of the spherical shell of radius r , Then electric flux linked with the sphere is $\phi = \oint E\, ds$ (here E and ds is parallel to each other as shown in figure)

But here , $E = q/4\pi\epsilon_0 r^2$

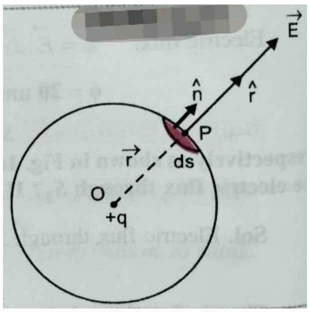
So, $\phi = \oint E\, ds = \oint(\frac{q}{4\pi\epsilon 0 r2})\, ds = (q/4\pi\epsilon_0{}^2) \oint ds = (q/4\pi\epsilon_0 r^2)\, 4\pi r^2 = q/\epsilon_0$

i.e. $\phi = \oint E\, ds = q/\epsilon_0$ Hence prove .

If the medium surrounding the charge has a dielectric constant 'K' then the flux

$\phi = q/K\epsilon_0$

Deduction of Coulomb's law on the basis of Gauss's law – Suppose a positive point charge is at O . We imagine a sphere of radius r with centre O . The magnitude of electric field intensity at that any point the surface of the sphere is E ,and it directed radially outward . The direction of the small area ds at the point p is along the direction of electric field E . i.e. electric field and area vector are parallel to each other



According to Gauss's theorem ,

$\phi = \oint E\, ds = q/\epsilon_0$

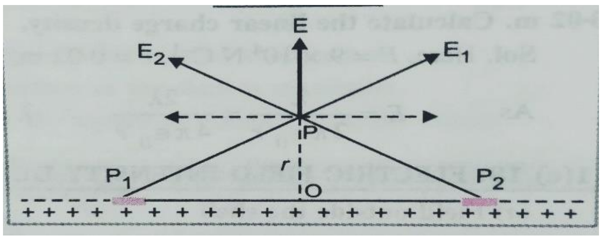
or , $\oint E\, ds \cos 0^0 = q/\epsilon_0$,

or, $E \oint ds = q/\epsilon_0$ or, $E\,(4\pi r^2) = q/\epsilon_0$

or, $E = q/4\pi r^2 \epsilon_0 = q/4\pi\epsilon_0 r^2$; This is the electric field intensity due to charge q at distance r , if we put a charge q_0 at point p then the force $F = q\, q_0 / 4\pi\epsilon_0 r^2$ which is coulomb's law .

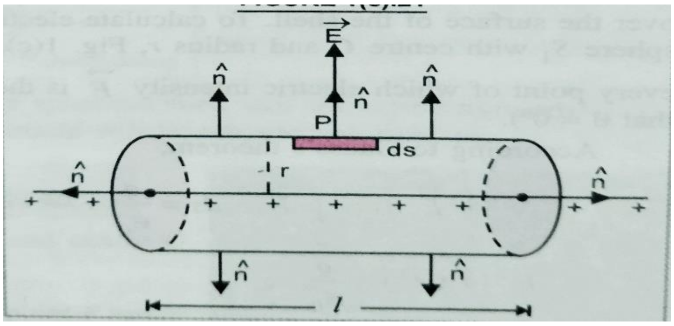
Application of Gauss's law -

Field due to an infinitely long straight uniformly charged wire – Consider an infinitely long thin wire having uniform charge density 'λ' , we have to find the electric field due to this charged wire at a point 'p' which is 'r' distance away from the wire . Let two points on the wire p_1 , p_2 of the wire at equal distance on either side of the point O as shown in figure.



Let , E_1 and E_2 are the electric field intensity it point P due to the charge on p_1 and p_2 . Then the net field will be E along the direction 'op' . Therefore total electric field at p will be radial .

Let us consider a right circular closed cylinder of radius r and length l as shown in figure



As we have proved above the electric field E is outward i.e. normal to the surface and area vector is same as E which is normal to the surface .

From Gauss's law , $\phi = \oint E\, ds$ = q/ϵ_0

Or , $\phi = \oint E\, ds\, \cos 0^0$ = q/ϵ_0

Or, E $\oint ds$ = q/ϵ_0

Or, E $2\pi rl = q/\epsilon_0$

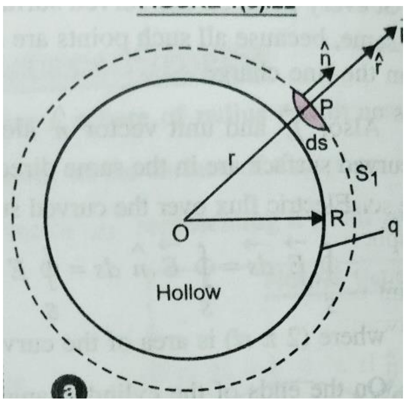
Or, $E = q/2\pi rl\epsilon_0 = \lambda/2\pi r\epsilon_0$ (since $q/l = \lambda$= linear charge density .)

So we can say Electric field $E \propto 1/r$; If $\lambda > 0$ then the electric field is radially outward

If $\lambda < 0$ then the electric field is radially inward .

Electric field intensity due to a charged spherical shell –

Case I – Field outside the shell - suppose a thin spherical shell is of radius R with centre O . let charge +q is given to the surface of the shell , which will spread over the surface of the shell uniformly we have to find the field at point p which is outside the sphere (r>R) . We draw a Gaussian surface as shown in figure (a)



.

Consider a small element on the Gaussian surface ds . the direction of field E and area vector are same which is radially outward as shown .

According to Gauss's theorem ,

$\phi = \oint E\, ds$ = q/ϵ_0

or , $\oint E\, ds$ cos0^0 = q/ϵ_0 ,

or, E $\oint ds$ = q/ϵ_0 or, E (4∏r^2) = q/ϵ_0

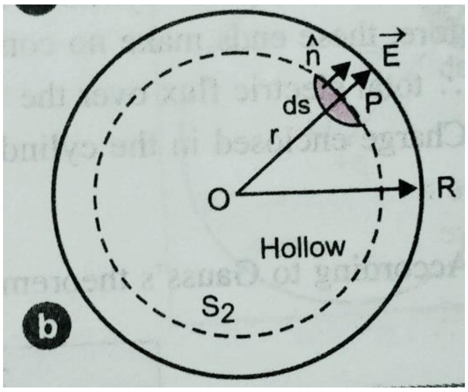
or, E= q/4∏r^2 ϵ_0 = q/4∏ϵ_0 r^2 ;

As shown in the result , for the points outside the shell charged spherical shell behaves like a point source at the centre .

If the points lie on the surface of the shell (r=R) then

E= q/4∏ϵ_0 R^2 = σ/ϵ_0 [where σ(surface charge density) = q/4∏r^2] .

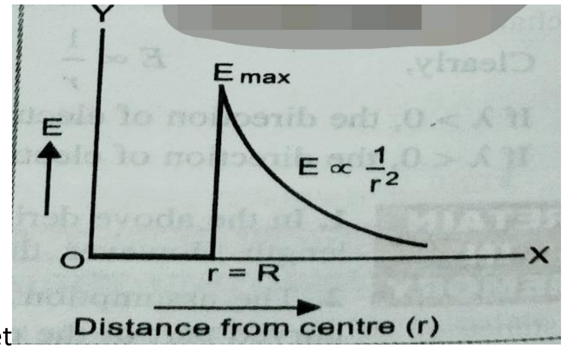
Case-ii – Field inside the spherical shell - - suppose a thin spherical shell is of radius R with centre O . let charge +q is given to the surface of the shell , which will spread over the surface of the shell uniformly we have to find the field at point p which is inside the sphere (r<R) . We draw a Gaussian surface as shown in figure(b)



According to Gauss's theorem ,

$\phi = \oint E\, ds$ = q/ϵ_0

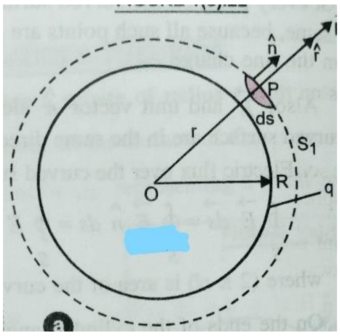
or , $\oint E\, ds$ cos0^0 = q/ϵ_0 But q=o (because total charge on the surface and no charge inside the Gaussian surface , so E = o , i.e there is no electric field inside the shell .

If we plot the graph field v/s distance of point we get 

Electric field intensity due to a nonconducting charged solid sphere -

let us consider a non-conducting solid sphere of radius R with centre O has uniform volume charge density ϱ . We have to calculate the electric field intensity at a point p due to this charged solid sphere .

Case-I – when point p is outside the sphere - suppose a solid spherical is of radius R with centre O . let charge +q is given to the surface of the sphere , which will spread uniformly throughout volume of the sphere we have to find the field at point p which is outside the sphere (r>R) . We draw a Gaussian surface as shown in figure (a)



Consider a small element on the Gaussian surface ds . the direction of field E and area vector are same which is radially outward as shown .

According to Gauss's theorem ,

$\phi = \oint E\, ds = q/\epsilon_0$ [since total charge given to the sphere is q]

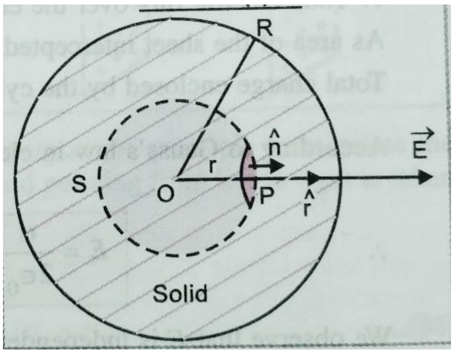
or , $\oint E\, ds \cos 0^0 = q/\epsilon_0$,

or, $E \oint ds = q/\epsilon_0$ or, $E\,(4\pi r^2) = q/\epsilon_0$

or, $E = q/4\pi r^2\, \epsilon_0 = q/4\pi\epsilon_0\, r^2$;

As shown in the result , for the points outside the sphere charged solid spherical behaves like a point source at the centre .

Case-ii- Inside the sphere - Let p is the point inside the sphere (r<R) and we have to find the field intensity at that point , imagine a surface s of radius r which will act as Gaussian surface as shown in figure.



According to the Gauss law –

$\phi = \oint E\, ds = q/\epsilon_0$ ……………………………i

or , $\oint E\, ds \cos 0^0 = q'/\epsilon$, Here ,[$\epsilon = \epsilon_r\, \epsilon_0$ (ϵ is the permittivity in the medium and ϵ_r is the relative permittivity of the medium)]

or, $E \oint ds = q'/\epsilon$ or, $E\,(4\pi r^2) = q'/\epsilon$

or, $E = q'/4\pi r^2\, \epsilon = q'/4\pi\epsilon\, r^2$ ……………………ii

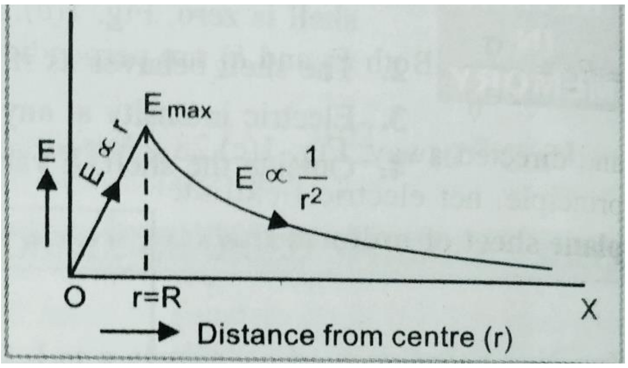
but $q' = (4/3)\, \pi r^3\, \rho$ ……………………….iii

from eq. ii and iii we get , $E = 4\pi r^3 \rho/3 \times 4\pi\epsilon r^2 = r\rho/3\epsilon$ (so , $E \alpha r$)

we can say that at the centre of the sphere r = 0 therefore at the centre E = 0).

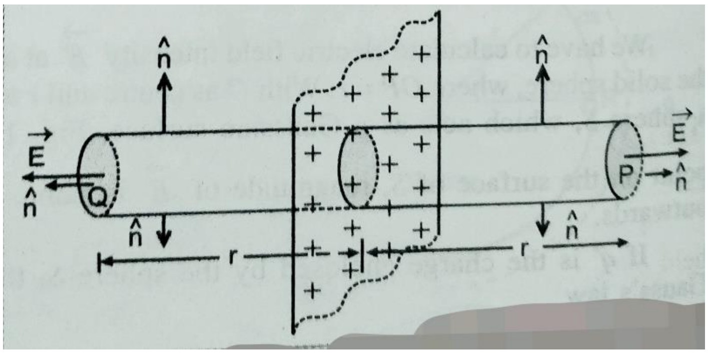
If r =R i.e. at the surface of the sphere $E = R\rho/3\epsilon$ [E = maximum].

If we plot the graph 'E' v/s 'r' we get the plot shown below –



Electric field intensity due to thin infinite charged plane sheet -

Suppose a thin infinite plane sheet has given charge q which spread throughout the surface uniformly . Therefore , surface charge density $\sigma = q/$ area of the sheet (A) . We have to find the electric field intensity at point p which is r distance away from the sheet .



From symmetry we find that p is either side of the sheet must be perpendicular to the plane of the sheet , having same magnitude at all points equidistant from the sheet .

Let us consider a cylinder of cross-sectional area ds around p is of length 2r , piercing through the sheet at the two edge of the cylinder p and q , here electric field is normal to the sheet which is radially outward .

Therefore electric flux over these edge $\phi = 2Eds$

According to the gauss law - $\phi = \oint E\, ds = q/\epsilon_0$

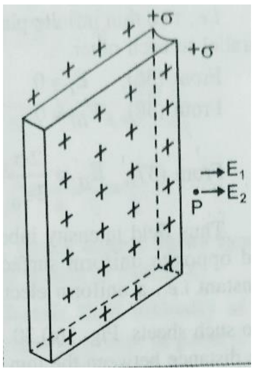
So ,$2 \oint E\, ds = q/\epsilon_0$

$2E \oint ds = 2E\, A = q/\epsilon_0$ Here A= area ;

So . $E = q/A2\epsilon_0$; but $q/A = \sigma$ so , $E = \sigma/2\epsilon_0$;

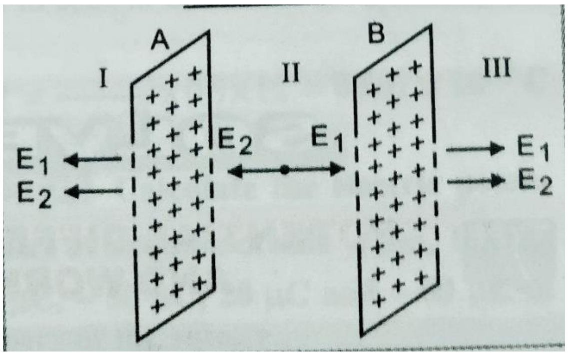
As we can see in the formula E is independent of r . If the sheet is positively charged , electric field is outward , and if sheet is negatively charged electric field is inward .

If the plane sheet has uniform thickness - If the sheet has uniform thickness in that case the surface charge density σ is uniform on both surfaces of the sheet . the electric field intensity E due to both surfaces at any point p will be same which is $E_1 = E_2 = \sigma/2\epsilon_0$ (Here both E_1 and E_2 both are perpendicular to the plane)



So net field at point p due to both the surfaces is given as , $E = E_1 + E_2 = \sigma/2\epsilon_0 + \sigma/2\epsilon_0 = 2\,\sigma/2\epsilon_0 = \sigma/\epsilon_0$

Electric field intensity due to two thin infinite parallel plane charged sheet - suppose A and B are two infinite charged plane sheet are placed parallel to each other . Suppose σ_1 and σ_2 are the charge density on plate A and B respectively . If E_1 and E_2 are the field at a point due to charged sheet A and B respectively .



Here, $E_1 = \sigma_1 / 2\epsilon_0$ and $E_2 = \sigma_2 / 2\epsilon_0$

So , net electric field ; in region (i) $E_I = - E_1 - E_2 = - (\sigma_1 /2\epsilon_0 + \sigma_2 /2\epsilon_0)$

In region (ii) $E_c = E_1 - E_2 = \sigma_1 /2\epsilon_0 - \sigma_2 /2\epsilon_0$

In region (iii) $E_r = E_1 + E_2 = \sigma_1 /2\epsilon_0 + \sigma_2 /2\epsilon_0$

Special case ;

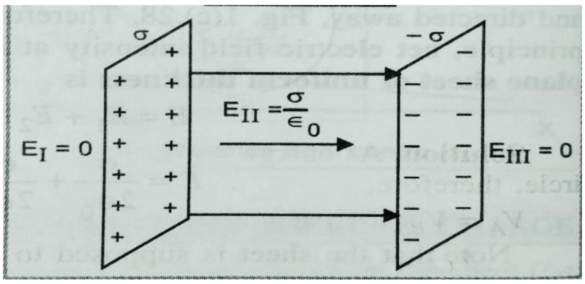
Case 1 - If $\sigma_1 = \sigma_2 = \sigma$

Then , $E_I = -\sigma/\epsilon_0$;

$E_{II} = 0$;

$E_{III} = \sigma/\epsilon_0$;

Again , case 2 - if $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$



Then , $E_I = 0$;

$E_{II} = \sigma/\epsilon_0$;

$E_{III} = 0$;