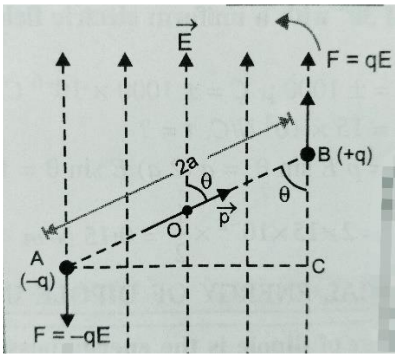
Expression for the torque experienced on dipole placed in uniform electric field –

Suppose an electric dipole AB having charges -q and +q are placed at distance 2a , and dipole moment 'p' is placed in an uniform electric field 'E' .



Then force experienced on both the charges are F = q E , but the direction of forces on each charges are opposite to each other . since the electric field E is constant then net force on the dipole will be zero (since forces are in opposite directions) .

Since the forces are equal , unlike and parallel acting at different points so , they form a couple . The couple tends to align the dipole axis along the direction of field 'E' .

Let's draw AC perpendicular on electric field 'E' .

Therefore torque acting on the dipole $\vec{\tau}$ = force x perpendicular distance

$\vec{\tau}$= F x AC ;

$\vec{\tau}$ = (q x E) x 2a sinΘ (AC = 2asinΘ) ; [where Θ is the angle between electric field and electric dipole]

$\vec{\tau}$= q 2a sinΘ x E = p E sinΘ;

$\vec{\tau} = \vec{p} \times \vec{E}$

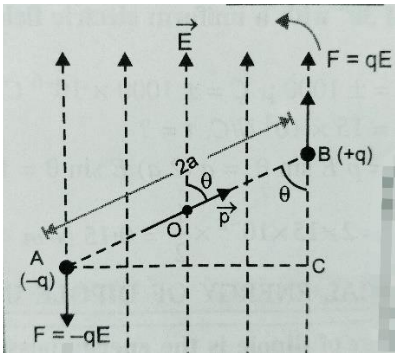
Special cases – Torque will be maximum when $\Theta=90^0$

Torque will be min when $\Theta= 0^0$ or 180^0

Potential energy of dipole is placed in uniform electric field -

Potential energy of a dipole is the energy possessed by the dipole by virtue of its particular position in the electric field .

Let an electric dipole of dipole moment 'p' is placed in uniform electric field 'E' at an angle 'Θ' . the torque experienced on the dipole $\vec{\tau}$ = p E sinΘ ,



So, work done in rotating the dipole through a small angle dΘ against the torque is

dW = τ x dΘ = pEsinΘ. dΘ ;

integrating both sides we get total work done from orientation Θ_1 to Θ_2

W = $\int_{\theta 1}^{\theta 2} \frac{pE\, sin\theta\, d\theta}{} =$ pE $[-\cos\Theta]_{\Theta 1}^{\Theta 2}$

W = - pE [cos Θ_2 -cos Θ_1]

So potential energy stored = work done = W = - pE [cos Θ_2 -cos Θ_1]

For single angle condition W = - pE cos Θ

Or , or U = -$\vec{p}\,.\vec{E}$

Special cases –

*When dipole is align in along the electric field i.e. $\Theta_1 = 0^0$ or $\Theta=0^0$ [stable equilibrium]

*when dipole is perpendicular to the electric field ; $\Theta= 90^0$ or U = -$\vec{p}\,.\vec{E}$ [un stable]

* when dipole is anti parallel to the electric field ; $\Theta = 180^0$ [unstable equilibrium]