Multiplication of vectors –

Scaler product (dot product) of –

When the product of two vector quantity gives a scaler quantity such kind of product is called scaler product or dot product. The dot product of two vectors $\vec{A}$ and $\vec{B}$ is represented by $\vec{A}.\vec{B}$,which is equal to the product of the magnitude of the two vectors $\vec{A}$ and $\vec{B}$ and cosine of the smaller angle between them .

(i) The scalar product of two vectors $\vec{a}$ and $\vec{b}$ is the scalar.

$$|\vec{a}||\vec{b}|\cos\theta$$

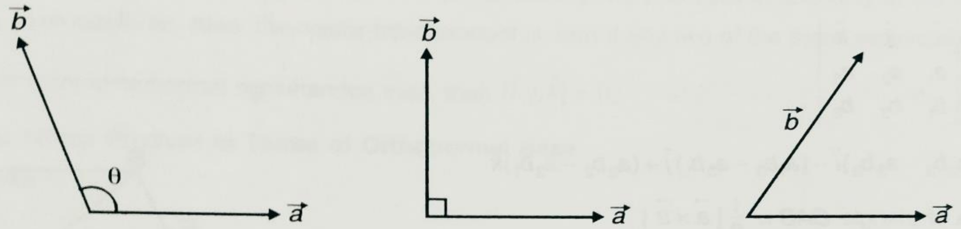
where θ is the angle between the vectors $\vec{a}$ and $\vec{b}$.

The scalar product of the vectors $\vec{a}$ and $\vec{b}$ is denoted by the symbol $\vec{a}.\vec{b}$ and due to this reason is also called the dot product of the two vectors.

Thus, we have by definition

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

(ii) θ being the angle between the vectors $\vec{a}$ and $\vec{b}$.



θ is acute $\Rightarrow \cos\theta > 0 \Rightarrow \vec{a}.\vec{b} > 0$

θ is right angle $\Rightarrow \cos\theta = 0 \Rightarrow \vec{a}.\vec{b} = 0$

θ is obtuse $\Rightarrow \cos\theta < 0 \Rightarrow \vec{a}.\vec{b} < 0$

(iii) If $\vec{a}$ be any vector then the scalar product $\vec{a}.\vec{a}$ of $\vec{a}$ with itself is given by

$\vec{a}.\vec{a} = |\vec{a}|^2$

Thus the length $|\vec{a}|$ of any vector is the non-negative square root $\sqrt{|\vec{a}|.|\vec{a}|}$ of the scalar product $\vec{a}.\vec{a}$.

Angle between two vectors in terms of scalar product is given by

$$\theta = \cos^{-1}\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

(iv) Scalar product is commutative

$\vec{a}.\vec{b} = \vec{b}.\vec{a}$

(v) Scalar multiplication of vectors is distributive over the addition of numbers

$\vec{a}.(\vec{b}+\vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

(vi) $(m\vec{a}).(n\vec{b}) = mn(\vec{a}.\vec{b})$

(vii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

Example of dot product - (i) work done W= $\vec{F}.\vec{S}$ (ii) power P= $\vec{F}.\vec{V}$ (iii) flux ϕ=$\vec{B}.\vec{A}$

Vector product (cross product)-

When the product of two vector quantities gives the vector quantity then such quantity is called vector product. The cross product of two vectors $\vec{A}$ and $\vec{B}$ is represented by $\vec{A}$ x $\vec{B}$,which is equal to the product of the magnitude of the two vectors $\vec{A}$ and $\vec{B}$ and sine of the smaller angle between them .the direction of resultant vector is perpendicular to both $\vec{A}$ and $\vec{B}$.

The vector product denoted by $\vec{a} \times \vec{b}$ of two vectors, $\vec{a}, \vec{b}$ taken in order, is the vector $\vec{c}$ where $\vec{c} = |\vec{a}||\vec{b}| \sin\theta \hat{n}$

θ being the angle between vectors, $0° \le \theta \le 180°$

The sense of $\vec{c}$ is such that the triad $\vec{a}, \vec{b}, \vec{c}$ forms a right-handed system.

(i) The vector product is not commutative. Indeed

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

(ii) $-\vec{a} \times \vec{b} = -(\vec{a} \times \vec{b})$

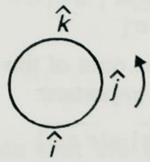
$$\vec{a} \times (-\vec{b}) = -(\vec{a} \times \vec{b})$$

$$(-\vec{a}) \times (-\vec{b}) = \vec{a} \times \vec{b}$$

(iii) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$$



In determinant form if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Then $\vec{a} \times \vec{b}$ is given by

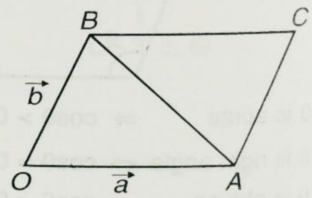
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_2b_2 - a_2b_1)\hat{k}$$

Area of triangle $OAB = \frac{1}{2}|\vec{a} \times \vec{b}|$

Area of parallelogram $OABC = |\vec{a} \times \vec{b}|$

where $\vec{a}$ and $\vec{b}$ are vectors corresponding to two adjacent sides.



Example-

(i)Torque – $\vec{\tau}$ = $\vec{F}$ X $\vec{R}$

(ii) angular velocity

(iii) linear velocity , etc.